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Applying the Theory of Characteristic Modes to the Analysis of Finite Antenna Array elements and Ground Planes of Finite Sizes

EuCAP '19, Krakow, Poland

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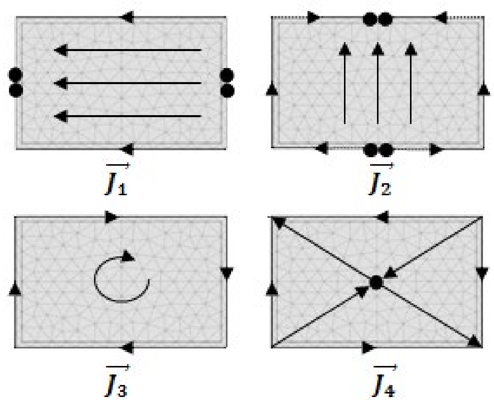
Faculty of Engineering

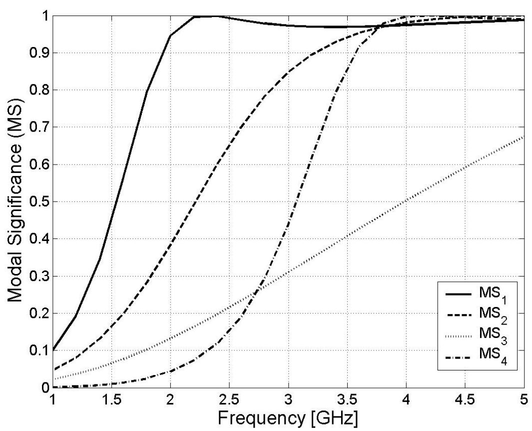
SKA AFRICA
SQUARE KILOMETRE ARRAY

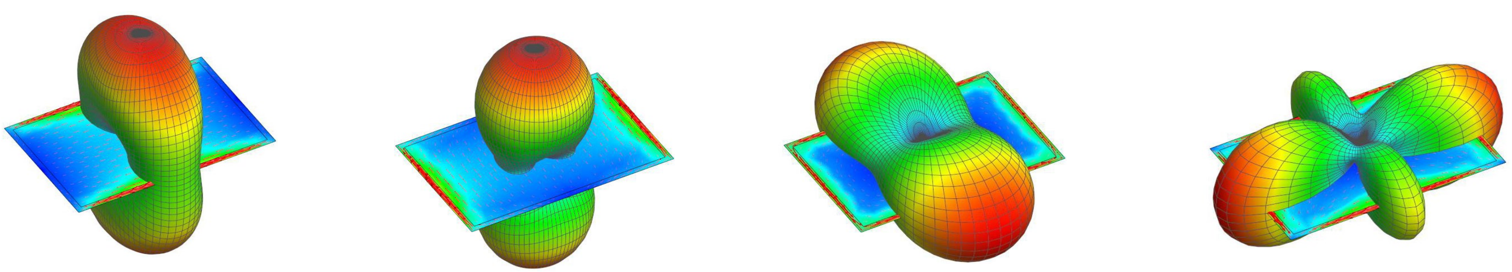
CMA

- CMA is a useful tool that can be used to optimise the design of various structures, e.g. for the excitation of particular modes to obtain a desired behaviour of an antenna.

- The modes are calculated by solving a generalized eigen-value equation $\mathbf{X}\mathbf{J}_n = \lambda_n \mathbf{R}\mathbf{J}_n$.

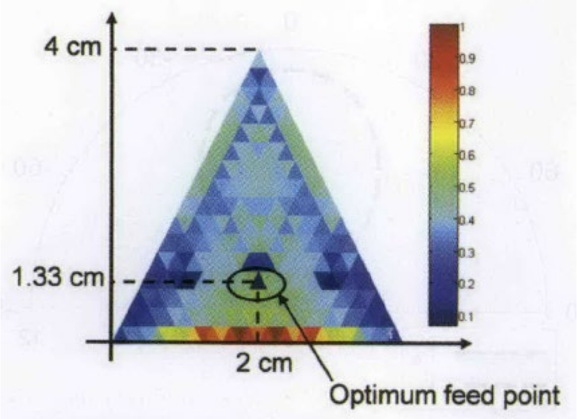


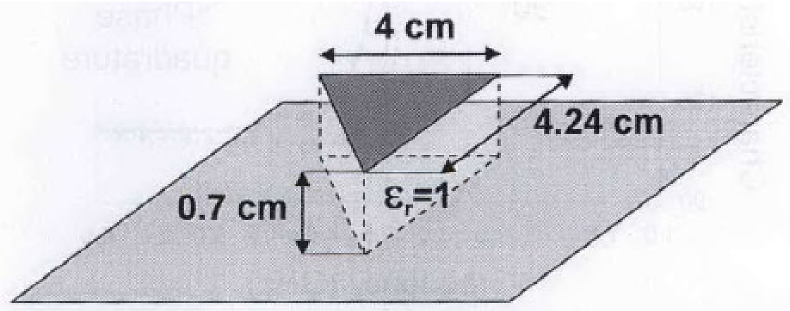




CMA

- CMA has been used to determine the optimal placement of a feed on a triangular patch antenna to obtain circular polarization

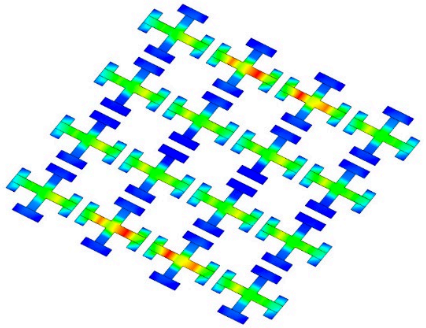




Ref: M. Cabedo-Fabres et al., "The Theory of Characteristic Modes Revisited: A Contribution to the Design of Antennas for Modern Applications," *IEEE Trans. Antennas and Propagation Magazine*, vol. 49, no. 5, pp. 52-68, October 2007

Motivation for this work

- When antennas are used in an array (typically above a ground plane), then their behavior can change as a result of mutual coupling.



- Goal here is to formulate a method that can be used to apply CMA to an antenna embedded in an array with a finite sized ground plane.

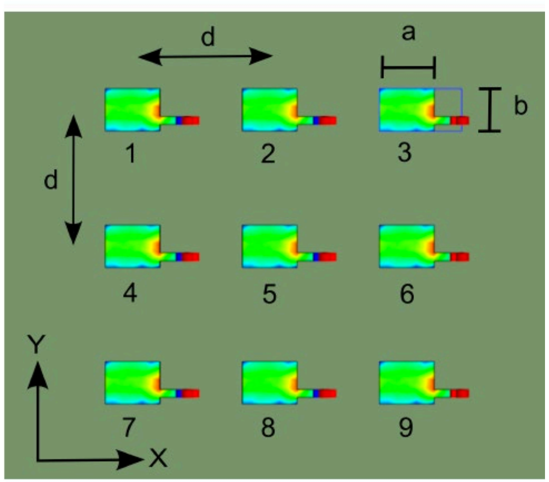
- Similar concept has been presented in 2012 by Ethier and McNamara, "*Sub-structure characteristic mode concept for antenna shape synthesis*," Electronics letters, vol. 48, no. 9, pp. 471–472, 2012.

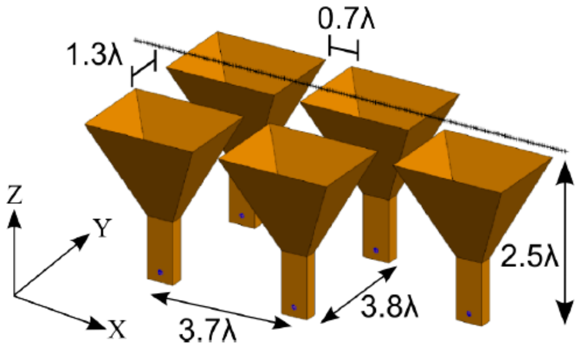
- CEM techniques applied here is the Domain Green's Function Method (**DGFM**) and the Numerical Green's Function (**NGF**) to extract a suitable matrix equation for the antenna on which CMA can be applied

Domain Green's Function Method (DGFM)

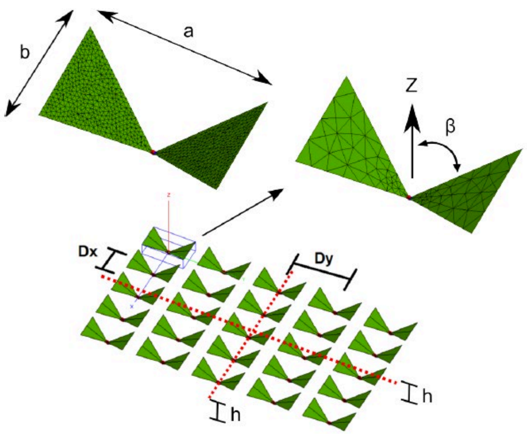
- A simple MoM matrix factorisation based approach, whereby the impedance matrix is factorised such that smaller matrix equations can be solved using LU factorisation

- Mutual coupling is accounted for using an active impedance matrix equation summation which is obtained after we do a block based factorisation of the MoM matrix:





(a) Horn anntenna array



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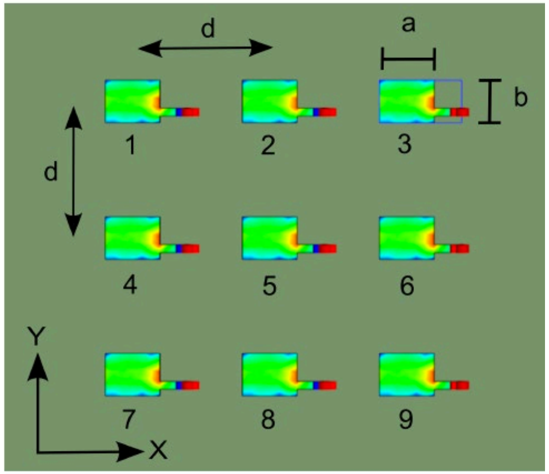

- Start with MoM: $\mathbf{ZJ} = \mathbf{V}$

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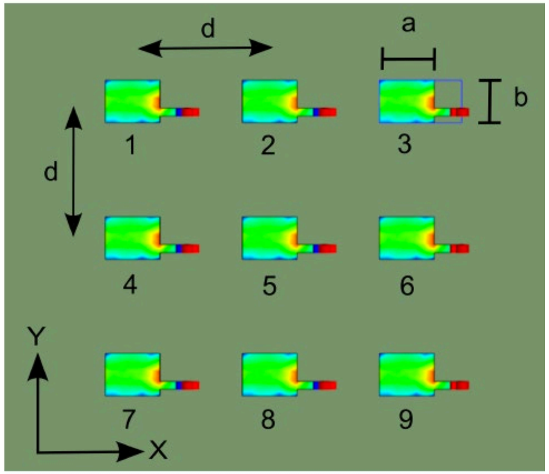

- Block based segmentation:

$$\begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \cdots & \mathbf{Z}_{1M} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \cdots & \mathbf{Z}_{2M} \\ \vdots & \ddots & \cdots & \vdots \\ \mathbf{Z}_{M1} & \cdots & \cdots & \mathbf{Z}_{MM} \end{bmatrix} \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \\ \vdots \\ \mathbf{J}_M \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \vdots \\ \mathbf{V}_M \end{bmatrix}$$

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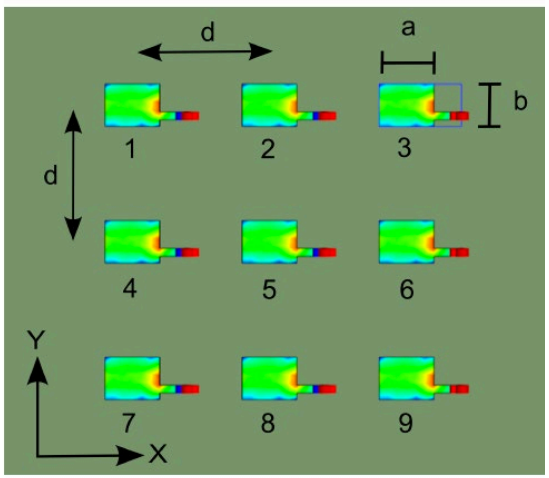

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- Assume similar currents (scaled by excitation coefficients

$$\mathbf{J}_2 = \alpha_{2p}\mathbf{J}_p; \quad \mathbf{J}_q = \alpha_{qp}\mathbf{J}_p; \quad \mathbf{J}_M = \alpha_{Mp}\mathbf{J}_p$$

$$\alpha_{qp} = \frac{V_q}{V_p}$$

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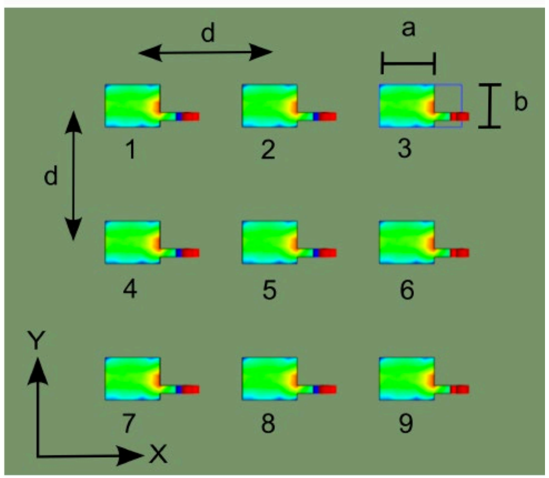

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- Derive active impedance matrix equation

$$\begin{aligned} \mathbf{Z}_i^{\mathrm{act}} &= \mathbf{Z}_{ii} + \alpha_{1i}\mathbf{Z}_{i1} + \alpha_{ki}\mathbf{Z}_{ik} + \ldots + \alpha_{Mi}\mathbf{Z}_{iM} \\ &= \sum_{k=1}^{M} \alpha_{ki}\mathbf{Z}_{ik}, \end{aligned}$$

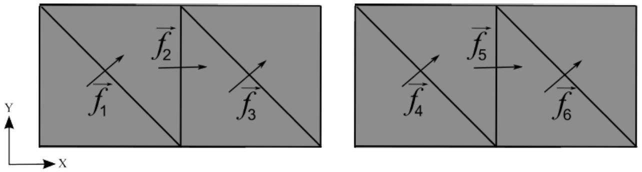
Domain Green's Function Method (DGFM)

- One way in which to improve the accuracy of the DGFM is to account for more accurate current distributions used for the initial "guess" using Jacobi iterations

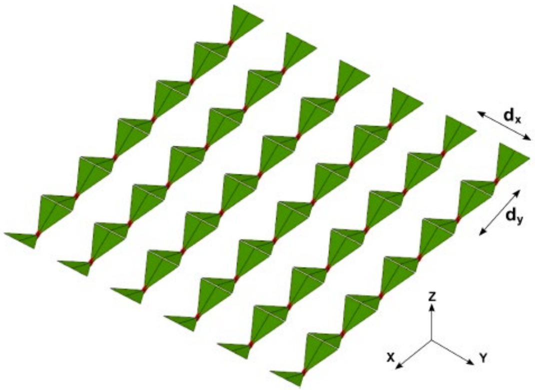
$$\mathbf{J}_q = \boldsymbol{\alpha}_{qp}\mathbf{J}_p.$$

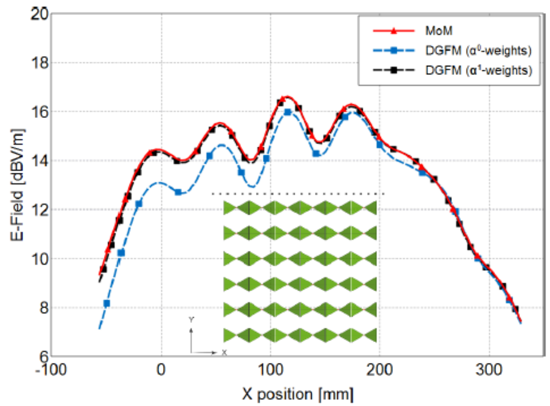
$$\mathbf{J}_p \simeq \mathbf{J}_{0p} - \sum_{m=1, m\neq p}^{M} \mathbf{Z}_{pp}^{-1}\mathbf{Z}_{pm}\mathbf{J}_{0m} \qquad \mathbf{J}_q \simeq \mathbf{J}_{0q} - \sum_{m=1, m\neq q}^{M} \mathbf{Z}_{qq}^{-1}\mathbf{Z}_{qm}\mathbf{J}_{0m}, \qquad \mathbf{J}_{0p} = (\mathbf{Z}_{pp})^{-1}\mathbf{V}_p$$

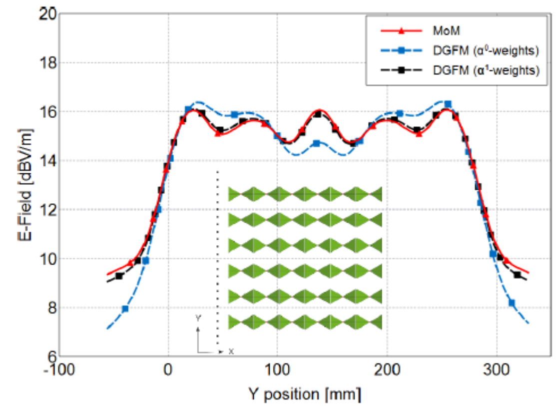
- In addition we also improve on the global scaling coefficients, to a localised scaling of each RWG



$$\boldsymbol{\alpha}_{qp} = \mathrm{diag}(I_1^q/I_1^p, I_2^q/I_2^p, \ldots, I_N^q/I_N^p). \qquad \mathbf{Z}_p^{\mathrm{act}} = \sum_{m=1}^{M} \mathbf{Z}_{pm}\boldsymbol{\alpha}_{mp}.$$







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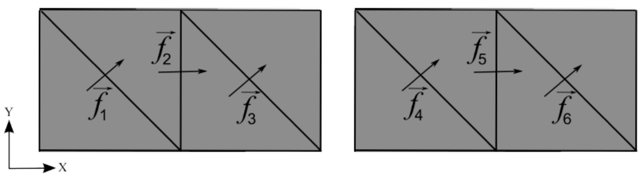
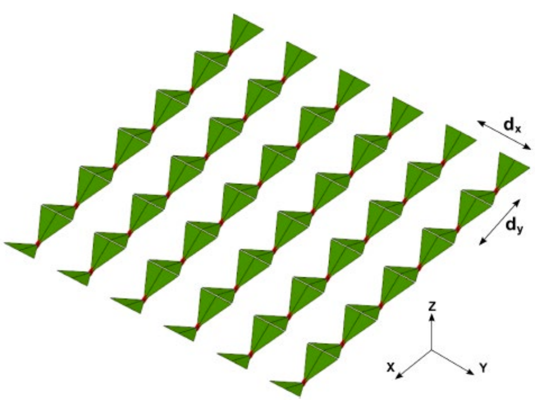
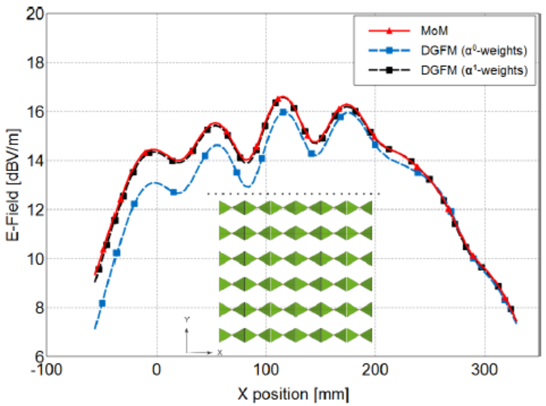
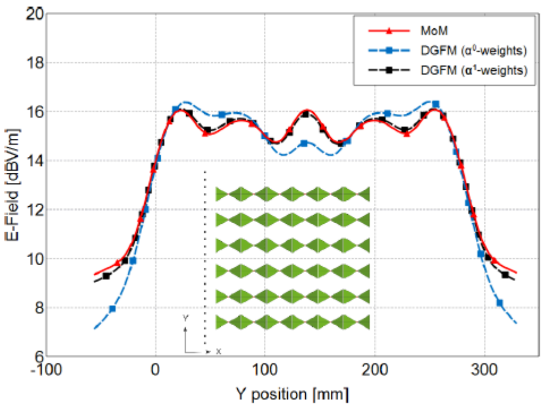


$$\boldsymbol{\alpha}_{qp} = \mathrm{diag}(I_1^q/I_1^p, I_2^q/I_2^p, \ldots, I_N^q/I_N^p). \qquad \boxed{\mathbf{Z}_p^{\mathrm{act}} = \sum_{m=1}^{M} \mathbf{Z}_{pm}\boldsymbol{\alpha}_{mp}.}$$







Including the NGF

- What happens when we include also now a ground plane?

- Split the problem into 2 domains, viz. the static and dynamic parts:

  $\Omega_A \rightarrow$ ground plane

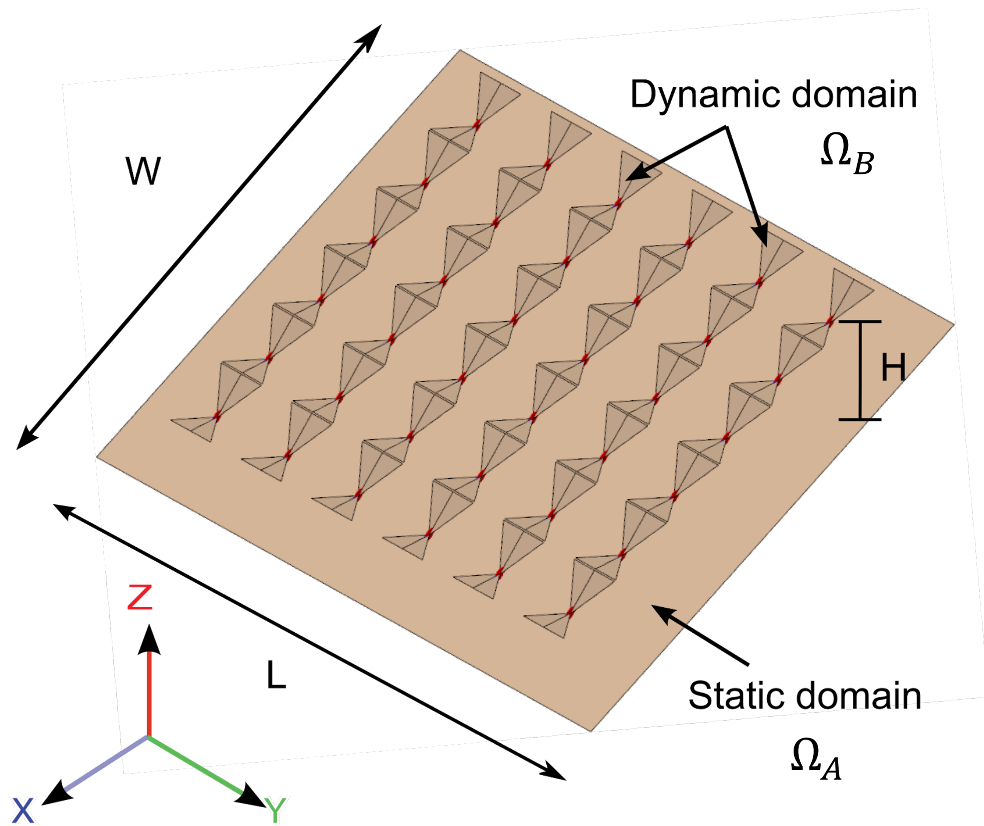
  $\Omega_B \rightarrow$ finite array

- Consider that the MoM eq. for the combined problem can now be written as follows:

$$\mathbf{ZJ} = \mathbf{V}$$

$$\downarrow$$

$$\begin{bmatrix} \mathbf{Z}_{\Omega_{AA}} & \mathbf{Z}_{\Omega_{AB}} \\ \mathbf{Z}_{\Omega_{BA}} & \mathbf{Z}_{\Omega_{BB}} \end{bmatrix} \begin{bmatrix} \mathbf{J}_{\Omega_A} \\ \mathbf{J}_{\Omega_B} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{\Omega_A} \\ \mathbf{V}_{\Omega_B} \end{bmatrix}$$



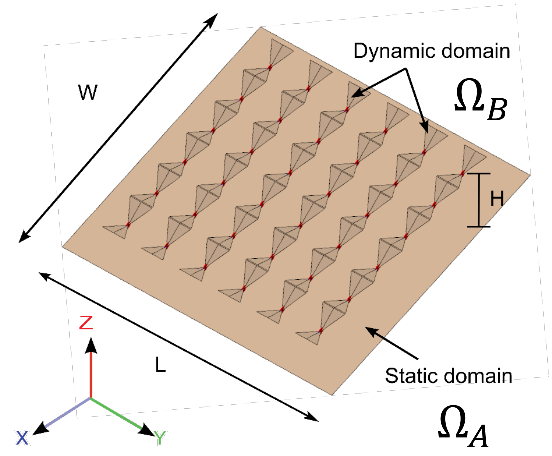
Including the NGF

- This factorization can be expanded:

$$\begin{bmatrix} \mathbf{Z}_{\Omega_{AA}} & \mathbf{Z}_{\Omega_{AB}} \\ \mathbf{Z}_{\Omega_{BA}} & \mathbf{Z}_{\Omega_{BB}} \end{bmatrix} \begin{bmatrix} \mathbf{J}_{\Omega_A} \\ \mathbf{J}_{\Omega_B} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{\Omega_A} \\ \mathbf{V}_{\Omega_B} \end{bmatrix}$$

$$\downarrow$$

$$\mathbf{Z}_{\Omega_{AA}} \mathbf{J}_{\Omega_A} + \mathbf{Z}_{\Omega_{AB}} \mathbf{J}_{\Omega_B} = \mathbf{0}$$
$$\mathbf{Z}_{\Omega_{BA}} \mathbf{J}_{\Omega_A} + \mathbf{Z}_{\Omega_{BB}} \mathbf{J}_{\Omega_B} = \mathbf{V}_{\Omega_B}$$



- The current on the **ground plane** (Ω_A) can be calculated as follows:

$$\begin{aligned} \mathbf{J}_{\Omega_A} &= \mathbf{Z}^{-1}_{\Omega_{AA}} (\mathbf{V}_{\Omega_A} - \mathbf{Z}_{\Omega_{AB}} \mathbf{J}_{\Omega_B}) \\ &= -\mathbf{Z}^{-1}_{\Omega_{AA}} \mathbf{Z}_{\Omega_{AB}} \mathbf{J}_{\Omega_B}. \end{aligned}$$

- The current on the **array** (Ω_B) can be calculated as follows:

$$\begin{aligned} \mathbf{J}_{\Omega_B} &= \mathbf{V}_{\Omega_B} (\mathbf{Z}_{\Omega_{BB}} - \mathbf{Z}_{\Omega_{BA}} \mathbf{Z}^{-1}_{\Omega_{AA}} \mathbf{Z}_{\Omega_{AB}})^{-1} \\ &= \mathbf{V}_{\Omega_B} (\zeta_{\Omega_{BB}})^{-1}, \end{aligned}$$

Including the NGF

- This factorization can be expanded:

$$\begin{bmatrix} \mathbf{Z}_{\Omega_{AA}} & \mathbf{Z}_{\Omega_{AB}} \\ \mathbf{Z}_{\Omega_{BA}} & \mathbf{Z}_{\Omega_{BB}} \end{bmatrix} \begin{bmatrix} \mathbf{J}_{\Omega_A} \\ \mathbf{J}_{\Omega_B} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{\Omega_A} \\ \mathbf{V}_{\Omega_B} \end{bmatrix}$$

$$\downarrow$$

$$\mathbf{Z}_{\Omega_{AA}} \mathbf{J}_{\Omega_A} + \mathbf{Z}_{\Omega_{AB}} \mathbf{J}_{\Omega_B} = \mathbf{0}$$
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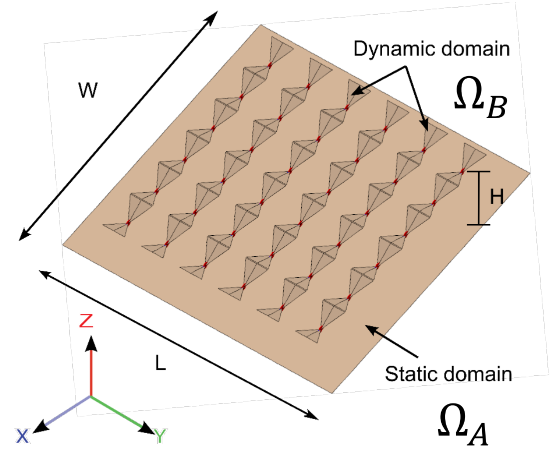


- The current on the **ground plane** (Ω_A) can be calculated as follows:

$$\begin{aligned} \mathbf{J}_{\Omega_A} &= \mathbf{Z}_{\Omega_{AA}}^{-1} (\mathbf{V}_{\Omega_A} - \mathbf{Z}_{\Omega_{AB}} \mathbf{J}_{\Omega_B}) \\ &= -\mathbf{Z}_{\Omega_{AA}}^{-1} \mathbf{Z}_{\Omega_{AB}} \mathbf{J}_{\Omega_B}. \end{aligned}$$

- The current on the **array** (Ω_B) can be calculated as follows:

$$\begin{aligned} \mathbf{J}_{\Omega_B} &= \mathbf{V}_{\Omega_B} (\mathbf{Z}_{\Omega_{BB}} - \mathbf{Z}_{\Omega_{BA}} \mathbf{Z}_{\Omega_{AA}}^{-1} \mathbf{Z}_{\Omega_{AB}})^{-1} \\ &= \mathbf{V}_{\Omega_B} \boxed{(\zeta_{\Omega_{BB}})}^{-1}, \end{aligned}$$

Including the NGF

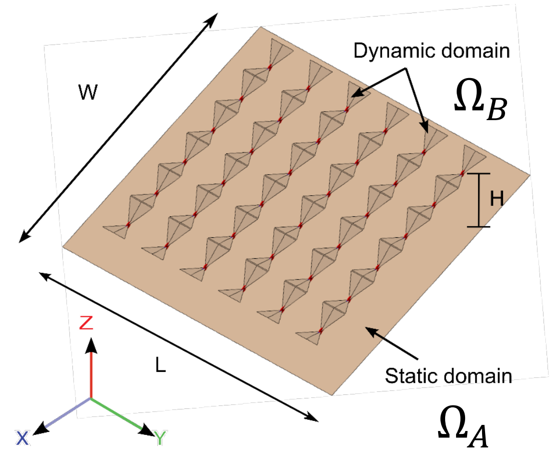
- For the array, we can now formulate a concise MoM eq. that includes the effect of the ground plane:

$$(\mathbf{Z}_{\Omega_{\mathrm{BB}}} - \mathbf{Z}_{\Omega_{\mathrm{BA}}}\mathbf{Z}_{\Omega_{\mathrm{AA}}}^{-1}\mathbf{Z}_{\Omega_{\mathrm{AB}}})\mathbf{J}_{\Omega_{\mathrm{B}}} = \mathbf{V}_{\Omega_{\mathrm{B}}}$$

$$\zeta_{\Omega_{\mathrm{BB}}}\mathbf{J}_{\Omega_{\mathrm{B}}} = \mathbf{V}_{\Omega_{\mathrm{B}}}$$

$$\downarrow$$

$$\begin{bmatrix} \zeta_{11} & \zeta_{12} & \cdots & \zeta_{1\mathrm{M}} \\ \zeta_{21} & \zeta_{22} & \cdots & \zeta_{2\mathrm{M}} \\ \vdots & \vdots & \ddots & \vdots \\ \zeta_{\mathrm{M}1} & \zeta_{\mathrm{M}2} & \cdots & \zeta_{\mathrm{MM}} \end{bmatrix} \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \\ \vdots \\ \mathbf{J}_\mathrm{M} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \vdots \\ \mathbf{V}_\mathrm{M} \end{bmatrix}$$



Including the NGF + DGFM

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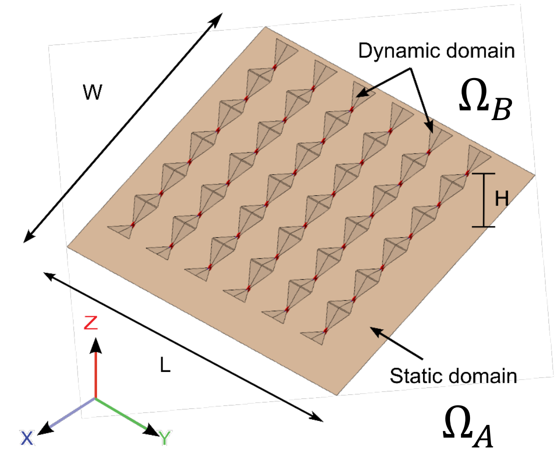

$$(\mathbf{Z}_{\Omega_{\mathrm{BB}}} - \mathbf{Z}_{\Omega_{\mathrm{BA}}}\mathbf{Z}_{\Omega_{\mathrm{AA}}}^{-1}\mathbf{Z}_{\Omega_{\mathrm{AB}}})\mathbf{J}_{\Omega_{\mathrm{B}}} = \mathbf{V}_{\Omega_{\mathrm{B}}}$$

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Can be reformulated using the DGFM:

$$\left[\sum_{m=1}^{M} \zeta_{\mathrm{pm}}\alpha_{mp}\right]\mathbf{J}_{\mathrm{p}} = \mathbf{V}_{\mathrm{p}}$$

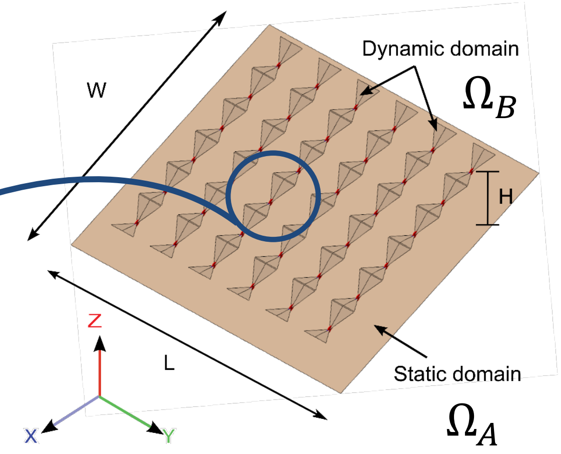
$$\mathbf{Z}_{\mathrm{p}}^{\mathrm{act}}\mathbf{J}_{\mathrm{p}} = \mathbf{V}_{\mathrm{p}}$$

with

$$\alpha_{qp} = \mathrm{diag}(I_1^q/I_1^p, I_2^q/I_2^p, ..., I_N^q/I_N^p)$$

Including the NGF + DGFM

- We have arrived now at our first objective, i.e. to formulate a MoM eq for the array element (embedded in the array environment and also taking into account the effect of the ground plane



$$\left[\sum_{m=1}^{M} \zeta_{\mathrm{pm}} \alpha_{mp}\right] \mathbf{J}_{\mathrm{p}} = \mathbf{V}_{\mathrm{p}}$$

$$\mathbf{Z}_{\mathrm{p}}^{\mathrm{act}} \mathbf{J}_{\mathrm{p}} = \mathbf{V}_{\mathrm{p}}$$

- We can now apply CMA on this modified impedance matrix:

$$\mathbf{X}_{\mathrm{p}}^{\mathrm{act}} \mathbf{J}_{\mathrm{np}} = \lambda_{\mathrm{np}} \mathbf{R}_{\mathrm{p}}^{\mathrm{act}} \mathbf{J}_{\mathrm{np}}$$

- As with conventional CMA we can calculate the *real* eigen-currents, $\mathbf{J}_{\mathrm{np}}$, and corresponding eigenvalues, λ_{np} by solving the eigenvalue eq.

- The eigen-currents are normalized as follows: $\langle \mathbf{J}_{\mathrm{np}}^{\dagger}, \mathbf{R}_{\mathrm{p}}^{\mathrm{act}} \mathbf{J}_{\mathrm{np}} \rangle = 1$

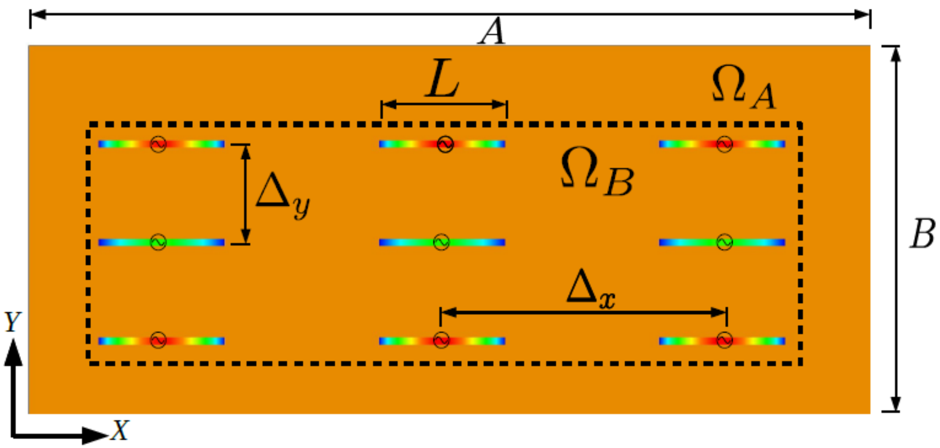
Example

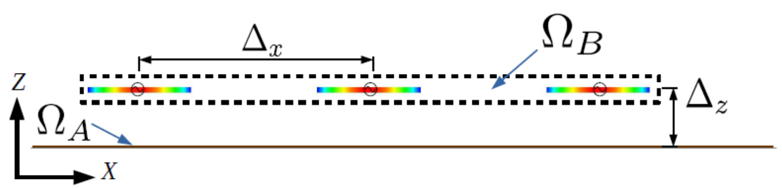
- Consider a 3x3 half-wavelength dipole antenna array with:

$$\Delta_x = \lambda,\ \Delta_y = 0.35\lambda$$

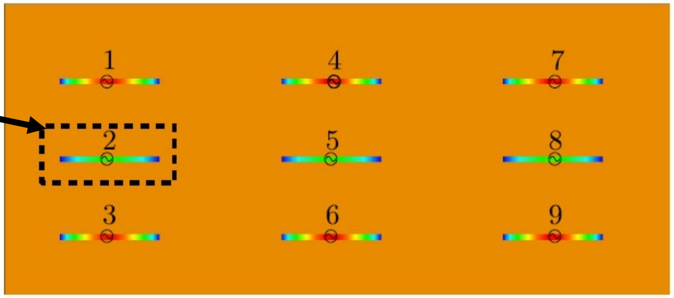
$$\Delta_z = 0.25\lambda,\ L = 0.5\lambda$$

$$A = 2.75\lambda \text{ and } B = 1.4\lambda$$





- Let's calculate the first 3 eigen-currents and eigenvalues for the for the 2nd array element



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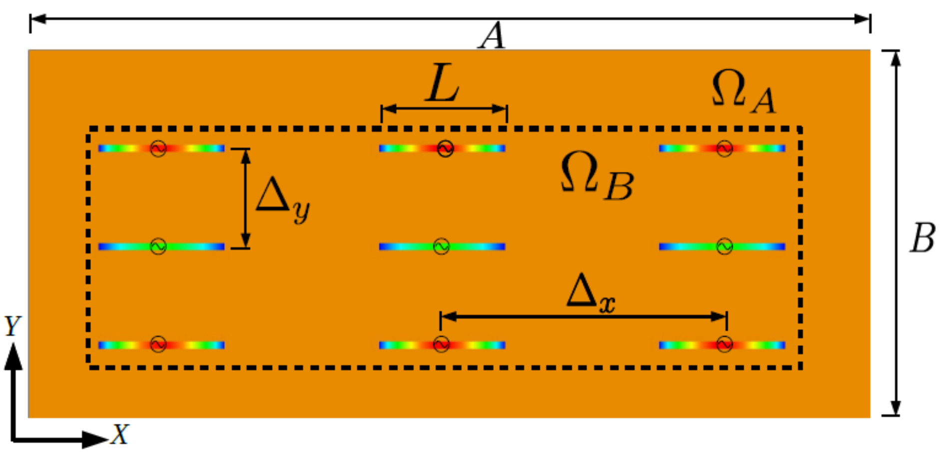
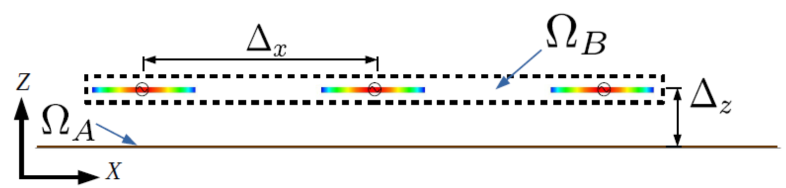
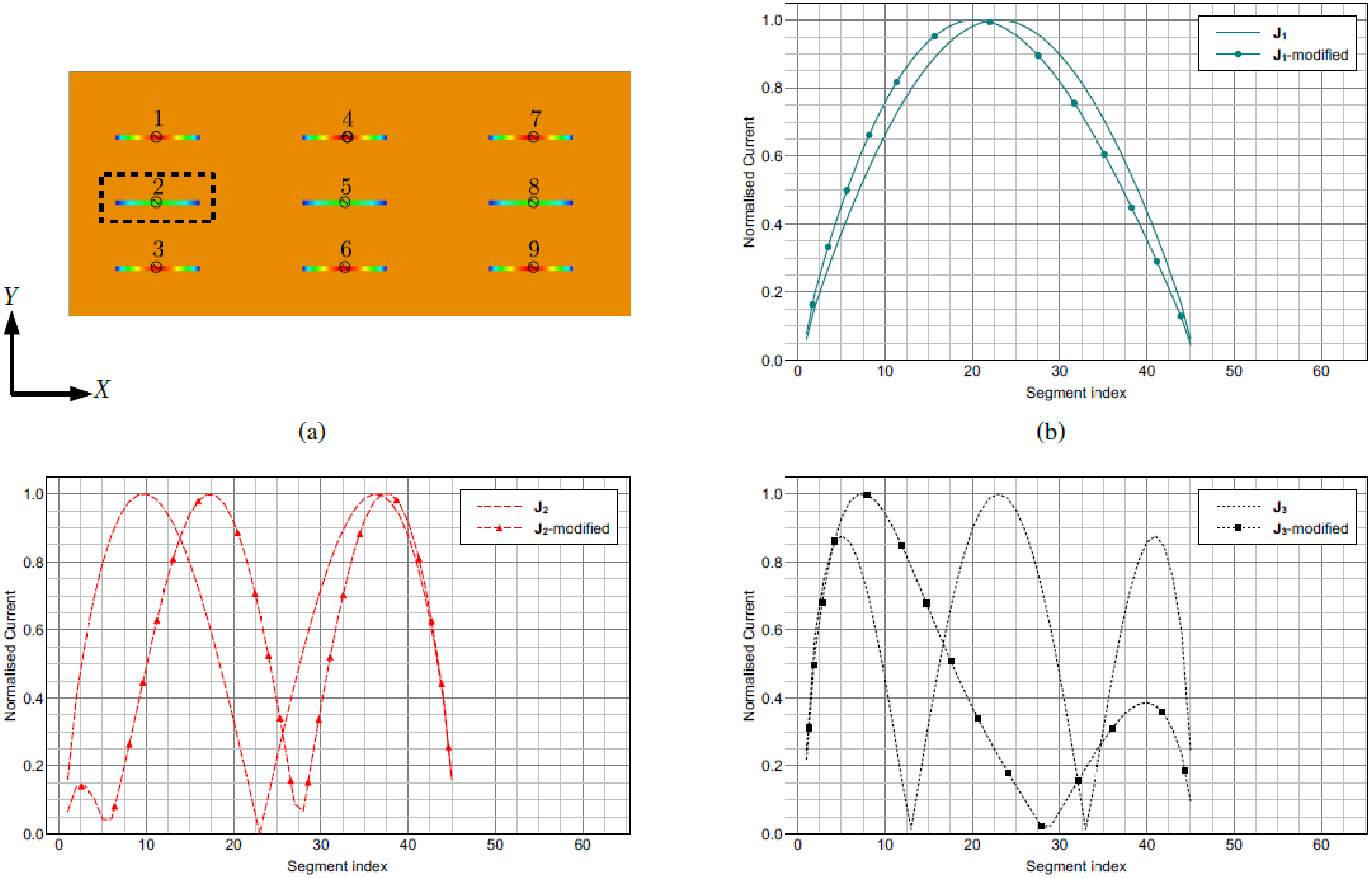




- The eigenvalues for the array elements that are obtained after solving their modified CMA equations clearly illustrate that the resonant properties of the modes differ

	Isolated case	$m = 1$	$m = 2$	$m = 4$	$m = 5$
λ_1	-5.60E-1	-1.28E-1	4.30E+0	-1.85E-1	8.13E+0
λ_2	-1.86E+2	-3.40E+2	1.09E+2	-4.90E+2	2.87E+2
λ_3	-1.52E+4	-3.31E+3	3.96E+2	-3.36E+3	4.86E+2

Example



(a) (b)

(c) (d)

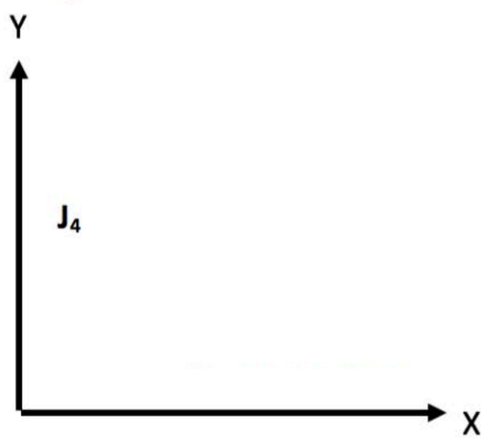
Example (square patches, no ground plane)

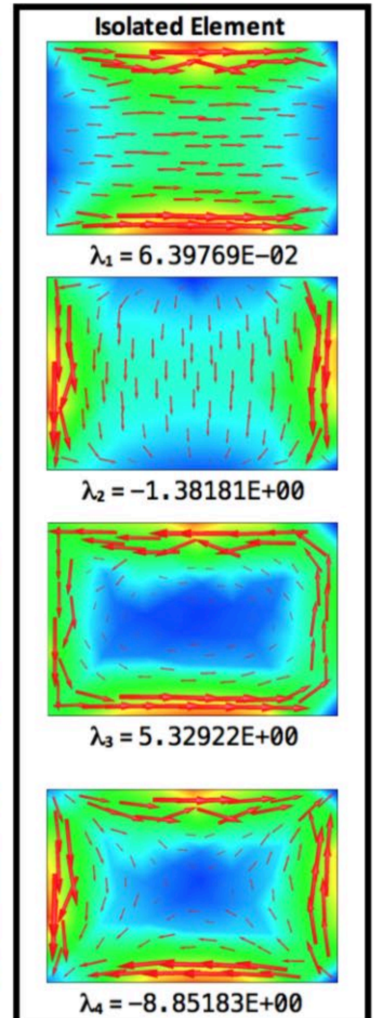
J_1

J_2

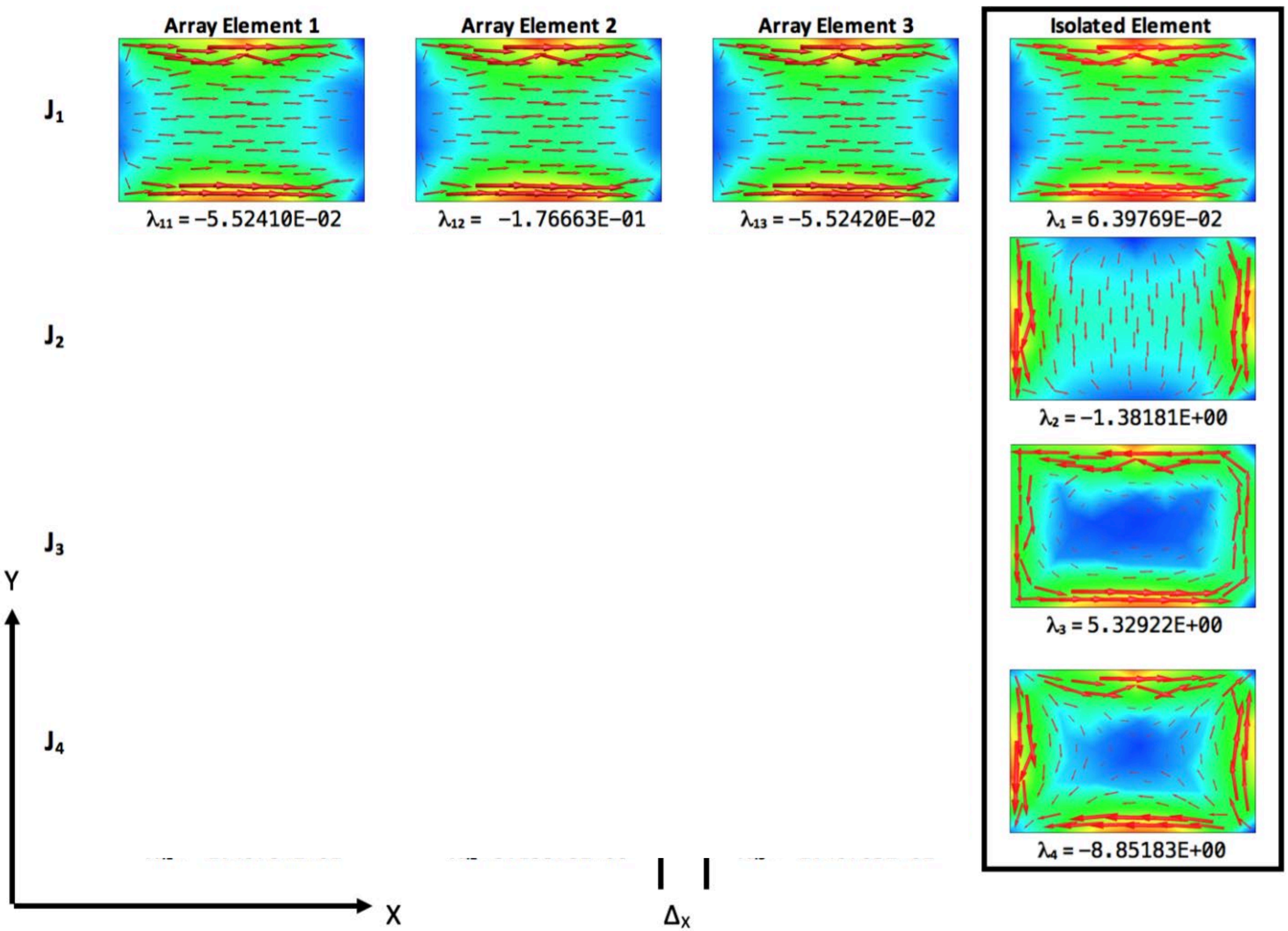
J_3

J_4

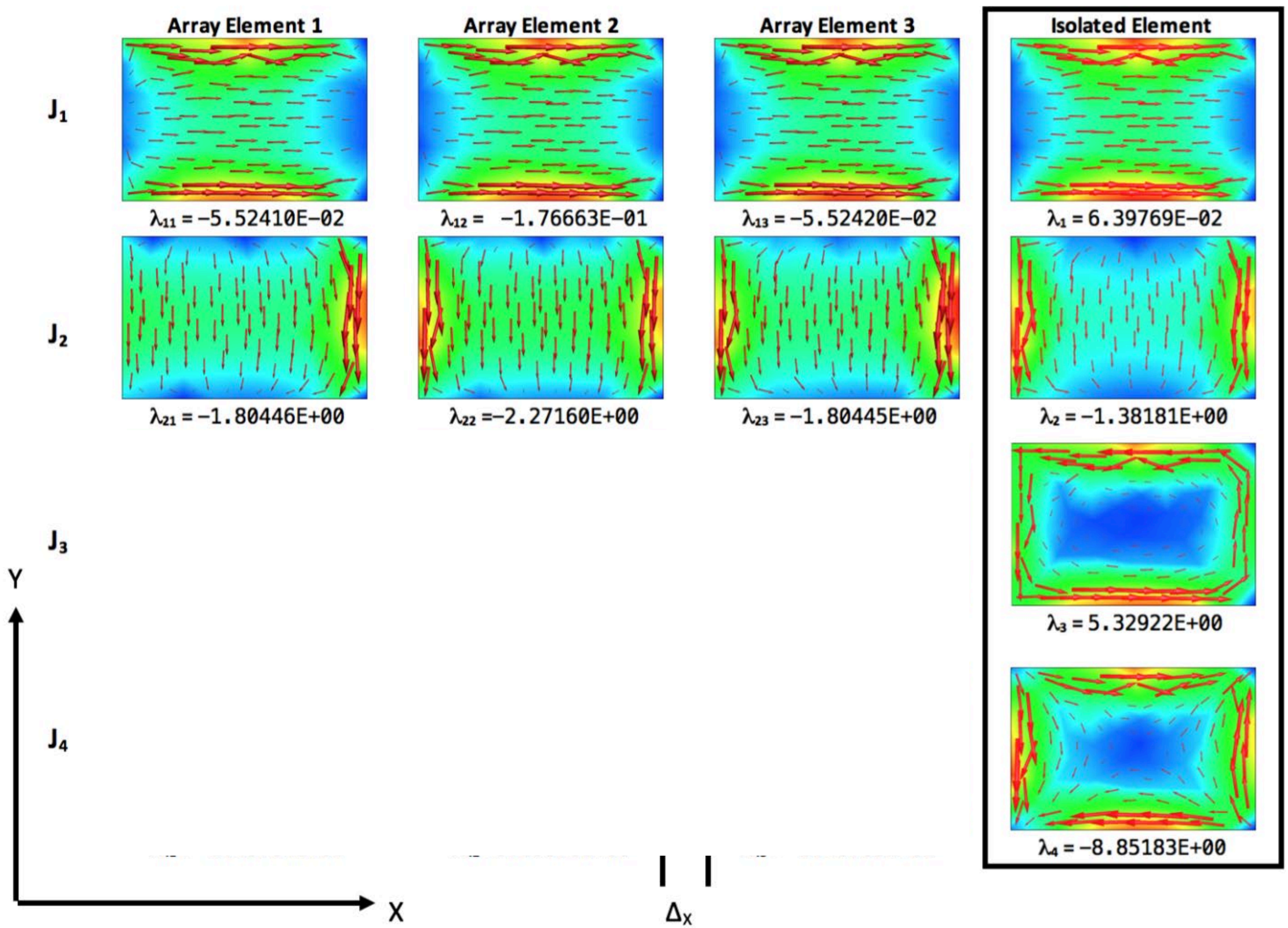




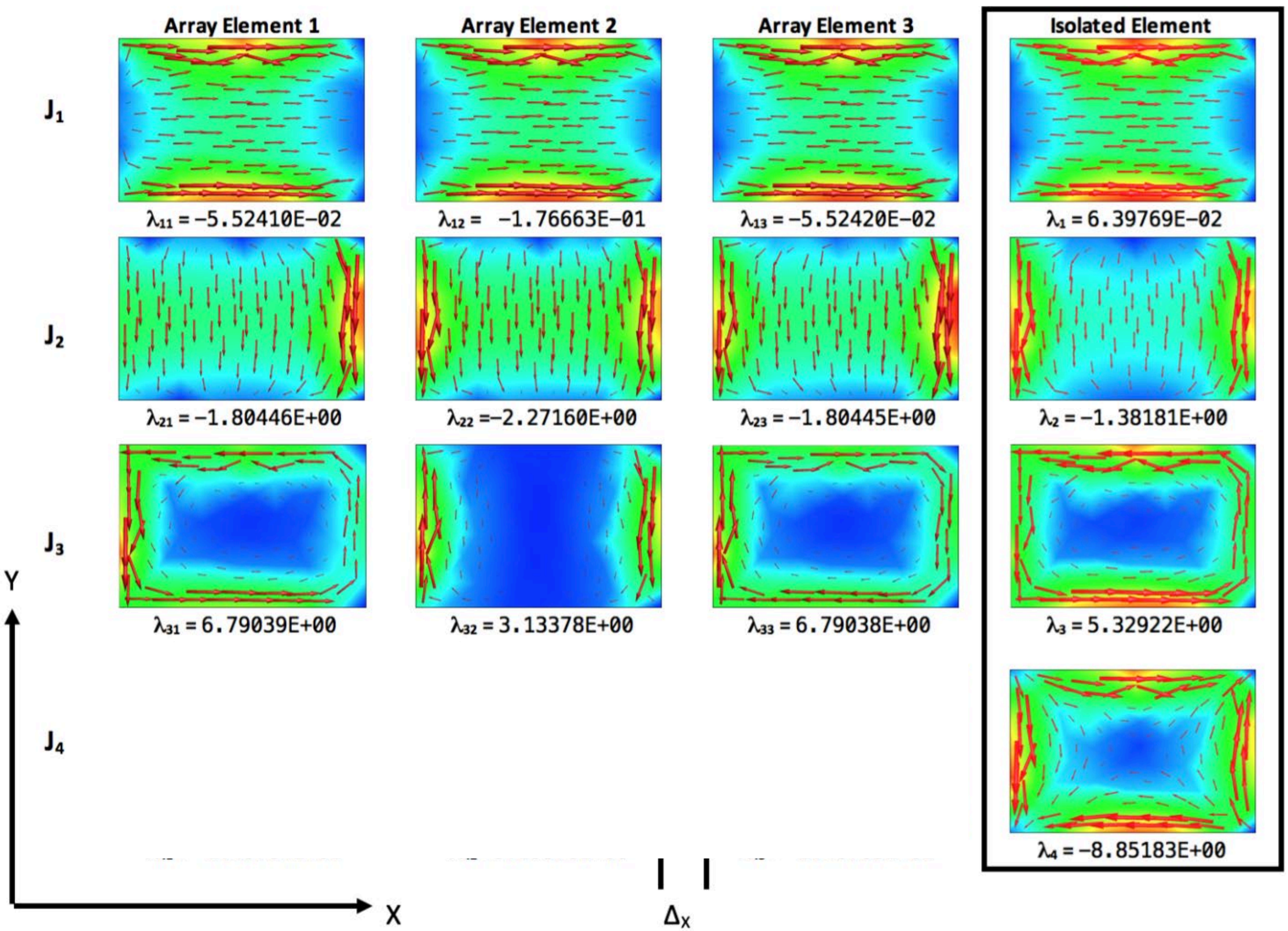
Example (square patches, no ground plane)



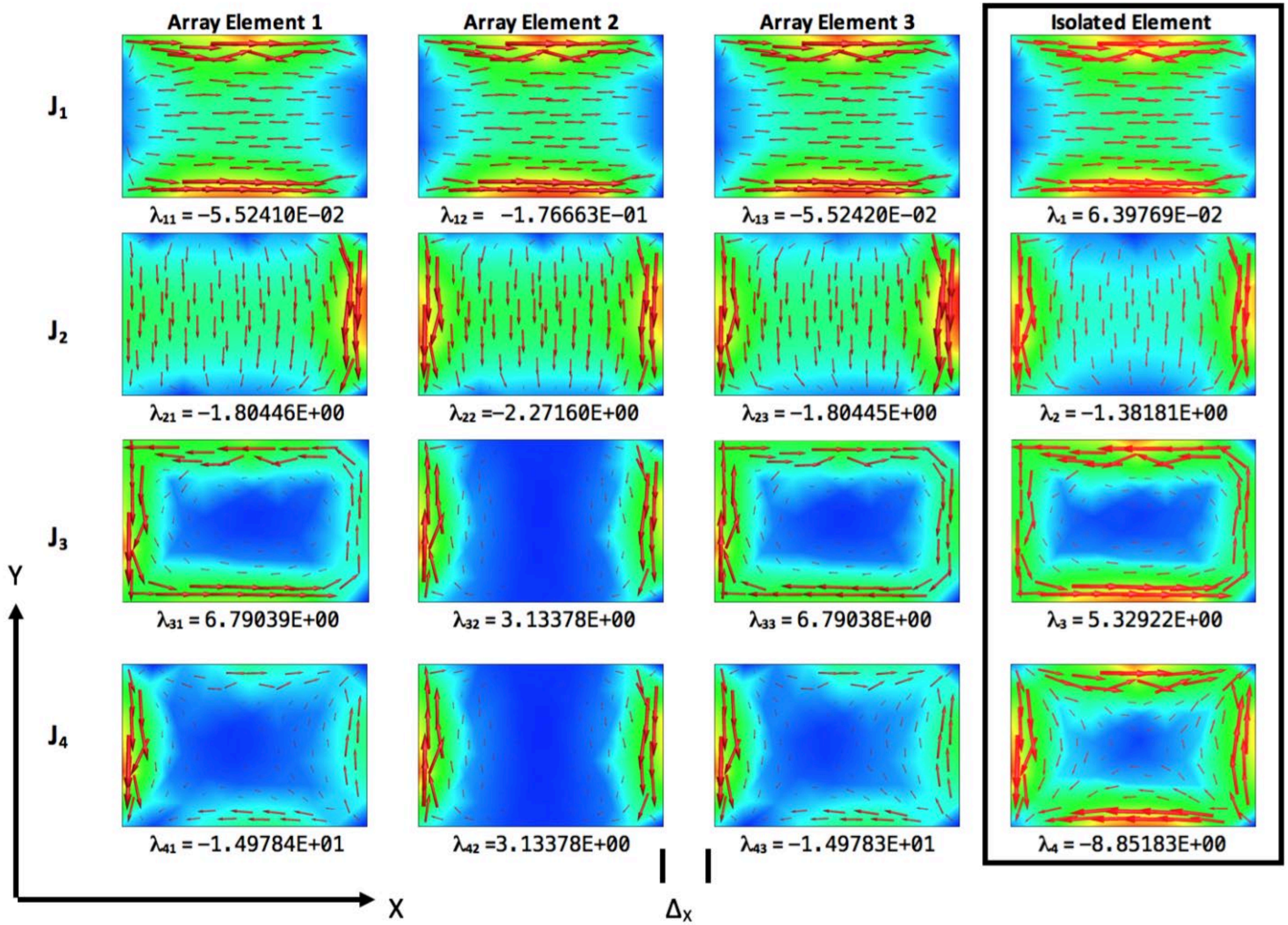
Example (square patches, no ground plane)



Example (square patches, no ground plane)



Example (square patches, no ground plane)

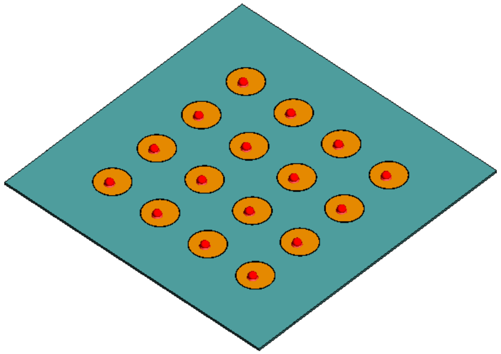


Conclusions & Way forward

- When using an element in an array (which might include a ground plane), mutual coupling can significantly effect the modal behaviour and should be taken into account when applying CMA.

- The DGFM allows us to model an active impedance matrix equation for each element that takes the array environment into account and by using the NGF we can include the effect of the ground plane.

- Future work includes applying this approach to find effective array feeding locations for finite arrays in the presence of ground planes



Acknowledgement

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Thank you!